

Question 5

Let the function $f(x)$ be defined as

$$f(x) = \begin{cases} kx^2 + 2x, & x < 3 \\ x^2 - 4kx + 9, & x \geq 3 \end{cases}$$

Find the value of k that makes $f(x)$ continuous at $x = 3$.

Question 5 - Solution

We are given:

$$f(x) = \begin{cases} kx^2 + 2x, & x < 3 \\ x^2 - 4kx + 9, & x \geq 3 \end{cases}$$

To ensure continuity at $x = 3$, the left-hand and right-hand limits must agree, and they must equal the function value:

$$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x) = f(3)$$

Step 1: Left-hand limit as $x \rightarrow 3^-$

From the first piece:

$$\lim_{x \rightarrow 3^-} f(x) = k(3)^2 + 2(3) = 9k + 6$$

Step 2: Right-hand limit and $f(3)$

From the second piece:

$$f(3) = (3)^2 - 4k(3) + 9 = 9 - 12k + 9 = 18 - 12k$$

Step 3: Set the two expressions equal for continuity

$$9k + 6 = 18 - 12k$$

Solve for k :

$$9k + 12k = 18 - 6 \quad 21k = 12 \quad k = \frac{12}{21} = \frac{4}{7}$$

Final Answer:

$$\boxed{k = \frac{4}{7}}$$