

Question 4

Consider the function

$$f(x) = \begin{cases} \frac{\sin(3x)}{x}, & x \neq 0 \\ c, & x = 0 \end{cases}$$

Find the value of c that makes $f(x)$ continuous at $x = 0$.

Question 4 - Solution

We are given:

$$f(x) = \begin{cases} \frac{\sin(3x)}{x}, & x \neq 0 \\ c, & x = 0 \end{cases}$$

We want $f(x)$ to be continuous at $x = 0$. This means:

$$\lim_{x \rightarrow 0} f(x) = f(0) = c$$

Step 1: Compute the limit as $x \rightarrow 0$

We use the standard limit identity:

$$\lim_{x \rightarrow 0} \frac{\sin(kx)}{x} = k$$

So, in our case:

$$\lim_{x \rightarrow 0} \frac{\sin(3x)}{x} = 3$$

Step 2: Set the limit equal to the function value

To ensure continuity at $x = 0$, we must have:

$$c = 3$$

Final Answer:

$$\boxed{c = 3}$$