

Question 3

Let the function $f(x)$ be defined by:

$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2}, & x \neq 2 \\ k, & x = 2 \end{cases}$$

Determine the value of k such that $f(x)$ is continuous at $x = 2$.

Question 3 - Solution

We are given:

$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2}, & x \neq 2 \\ k, & x = 2 \end{cases}$$

To make $f(x)$ continuous at $x = 2$, the following condition must hold:

$$\lim_{x \rightarrow 2} f(x) = f(2)$$

Step 1: Compute the limit as $x \rightarrow 2$

Factor the numerator:

$$x^2 - 4 = (x - 2)(x + 2)$$

For $x \neq 2$, the function simplifies to:

$$f(x) = x + 2$$

So,

$$\lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} (x + 2) = 4$$

Step 2: Match the function value

Continuity requires:

$$f(2) = k = 4$$

Final Answer:

$$\boxed{k = 4}$$