

Question 2

For real constants a and b , let

$$f(x) = \begin{cases} \frac{\sin(ax)}{x}, & x \neq 0, \\ b, & x = 0. \end{cases}$$

Classify the pairs (a, b) for which f is continuous at zero and those for which it has a removable discontinuity there.

State how to remove the discontinuity.

Question 2 - Solution

For $a \neq 0$, write

$$\frac{\sin(ax)}{x} = a \frac{\sin(ax)}{ax} \longrightarrow a.$$

For $a = 0$, the quotient is identically zero away from zero, so the same limit holds.

Thus $\lim_{x \rightarrow 0} f(x) = a$ for every real a .

Since $f(0) = b$, continuity holds exactly when $\boxed{b = a}$.

When $\boxed{b \neq a}$ the limit exists but differs from $f(0)$, so the discontinuity is removable by resetting $f(0) = a$.

A function cannot be both continuous and discontinuous at the same point.