

Question 9

Evaluate the limit:

$$\lim_{x \rightarrow -\infty} \frac{x^3 + 4x^2 - 7}{2x^3 - 5x + 1}$$

Question 9 - Solution

We are given the rational function:

$$\lim_{x \rightarrow -\infty} \frac{x^3 + 4x^2 - 7}{2x^3 - 5x + 1}$$

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Step 1: Analyze degrees of numerator and denominator

- The highest degree in both numerator and denominator is 3. - For limits at infinity, when the degrees match, the limit is the ratio of the leading coefficients.

So:

$$\lim_{x \rightarrow -\infty} \frac{x^3 + 4x^2 - 7}{2x^3 - 5x + 1} = \frac{1}{2}$$

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Step 2: Justify with algebraic manipulation (optional)

Divide numerator and denominator by x^3 :

$$= \frac{1 + \frac{4}{x} - \frac{7}{x^3}}{2 - \frac{5}{x^2} + \frac{1}{x^3}}$$

As $x \rightarrow -\infty$, the fractions go to 0:

$$\frac{1 + 0 - 0}{2 - 0 + 0} = \frac{1}{2}$$

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Final Answer:

$$\boxed{\frac{1}{2}}$$