

Question 6

Evaluate the limit:

$$\lim_{x \rightarrow -\infty} \left(\frac{3x^3 + 2x^2 - x + 5}{\sqrt{x^6 + x^2 + 1}} \right)$$

Question 6 - Solution

We are given:

$$\lim_{x \rightarrow -\infty} \left(\frac{3x^3 + 2x^2 - x + 5}{\sqrt{x^6 + x^2 + 1}} \right)$$

Step 1: Factor out highest powers

Numerator is a cubic polynomial: factor x^3 :

$$3x^3 + 2x^2 - x + 5 = x^3 \left(3 + \frac{2}{x} - \frac{1}{x^2} + \frac{5}{x^3} \right)$$

Denominator is a square root of a degree-6 polynomial:

$$\sqrt{x^6 + x^2 + 1} = |x^3| \sqrt{1 + \frac{1}{x^4} + \frac{1}{x^6}}$$

Note: Since $x \rightarrow -\infty$, we have $|x^3| = -x^3$

So,

$$\sqrt{x^6 + x^2 + 1} = -x^3 \sqrt{1 + \frac{1}{x^4} + \frac{1}{x^6}}$$

Step 2: Substitute into the expression

$$\frac{x^3 \left(3 + \frac{2}{x} - \frac{1}{x^2} + \frac{5}{x^3} \right)}{-x^3 \sqrt{1 + \frac{1}{x^4} + \frac{1}{x^6}}}$$

Cancel x^3 :

$$\frac{3 + \frac{2}{x} - \frac{1}{x^2} + \frac{5}{x^3}}{-\sqrt{1 + \frac{1}{x^4} + \frac{1}{x^6}}}$$

Step 3: Take the limit as $x \rightarrow -\infty$

All terms with $\frac{1}{x^n} \rightarrow 0$. So:

$$\lim_{x \rightarrow -\infty} \frac{3 + \frac{2}{x} - \frac{1}{x^2} + \frac{5}{x^3}}{-\sqrt{1 + \frac{1}{x^4} + \frac{1}{x^6}}} = \frac{3}{-1} = -3$$

Final Answer:

$$\boxed{-3}$$