

Question 5

Evaluate the limit:

$$\lim_{x \rightarrow \infty} \left(\frac{x^2 + 3x - 7}{\sqrt{x^4 + 2x^3 + 5}} \right)$$

Question 5 - Solution

We are given:

$$\lim_{x \rightarrow \infty} \left(\frac{x^2 + 3x - 7}{\sqrt{x^4 + 2x^3 + 5}} \right)$$

Step 1: Understand the growth of numerator and denominator

As $x \rightarrow \infty$, both numerator and denominator tend to infinity, so we need to simplify.

Step 2: Factor out highest power of x

In the numerator: highest power is x^2 In the denominator (inside square root): highest power is x^4

Factor numerator:

$$x^2 \left(1 + \frac{3}{x} - \frac{7}{x^2} \right)$$

Factor denominator inside square root:

$$\sqrt{x^4 \left(1 + \frac{2}{x} + \frac{5}{x^4} \right)} = x^2 \sqrt{1 + \frac{2}{x} + \frac{5}{x^4}}$$

Step 3: Substitute factored forms into the expression**

$$\frac{x^2 \left(1 + \frac{3}{x} - \frac{7}{x^2} \right)}{x^2 \sqrt{1 + \frac{2}{x} + \frac{5}{x^4}}}$$

Cancel x^2 :

$$\frac{1 + \frac{3}{x} - \frac{7}{x^2}}{\sqrt{1 + \frac{2}{x} + \frac{5}{x^4}}}$$

Step 4: Take the limit as $x \rightarrow \infty$

As $x \rightarrow \infty$, all terms with $\frac{1}{x}$ go to 0:

$$\lim_{x \rightarrow \infty} \frac{1 + \frac{3}{x} - \frac{7}{x^2}}{\sqrt{1 + \frac{2}{x} + \frac{5}{x^4}}} = \frac{1 + 0 - 0}{\sqrt{1 + 0 + 0}} = \frac{1}{1} = 1$$

Final Answer:

$$\boxed{1}$$