

Question 4

Evaluate the limit:

$$\lim_{x \rightarrow -\infty} \frac{x^3 + 2x^2 - 5x + 7}{x^3 - 4x + 1}$$

Question 4 - Solution

We are asked to evaluate:

$$\lim_{x \rightarrow -\infty} \frac{x^3 + 2x^2 - 5x + 7}{x^3 - 4x + 1}$$

Step 1: Compare the degrees of numerator and denominator

Both numerator and denominator are degree 3 polynomials.

When the degrees are the same, the limit as $x \rightarrow \pm\infty$ is the ratio of the leading coefficients.

Step 2: Extract leading terms

Numerator leading term: x^3 , Denominator leading term: x^3

So,

$$\lim_{x \rightarrow -\infty} \frac{x^3 + 2x^2 - 5x + 7}{x^3 - 4x + 1} = \lim_{x \rightarrow -\infty} \frac{1 + \frac{2}{x} - \frac{5}{x^2} + \frac{7}{x^3}}{1 - \frac{4}{x^2} + \frac{1}{x^3}}$$

As $x \rightarrow -\infty$, all terms with $\frac{1}{x^k} \rightarrow 0$ for $k > 0$

Step 3: Evaluate the limit

$$\lim_{x \rightarrow -\infty} \frac{1 + 0 - 0 + 0}{1 - 0 + 0} = \frac{1}{1} = 1$$

Final Answer:

$$\boxed{1}$$