

Question 3

Evaluate the limit:

$$\lim_{x \rightarrow \infty} \left(\sqrt{x^2 + 6x + 5} - \sqrt{x^2 + 2x + 1} \right).$$

Question 3 - Solution

We evaluate

$$\lim_{x \rightarrow \infty} \left(\sqrt{x^2 + 6x + 5} - \sqrt{x^2 + 2x + 1} \right).$$

This expression has the indeterminate form $\infty - \infty$, so we rationalize.

Step 1: Multiply by the Conjugate

$$= \lim_{x \rightarrow \infty} \left(\sqrt{x^2 + 6x + 5} - \sqrt{x^2 + 2x + 1} \right) \cdot \frac{\sqrt{x^2 + 6x + 5} + \sqrt{x^2 + 2x + 1}}{\sqrt{x^2 + 6x + 5} + \sqrt{x^2 + 2x + 1}}.$$

This simplifies to

$$= \lim_{x \rightarrow \infty} \frac{(x^2 + 6x + 5) - (x^2 + 2x + 1)}{\sqrt{x^2 + 6x + 5} + \sqrt{x^2 + 2x + 1}}.$$

Step 2: Simplify the Numerator

$$(x^2 + 6x + 5) - (x^2 + 2x + 1) = 4x + 4,$$

so the limit becomes

$$\lim_{x \rightarrow \infty} \frac{4x + 4}{\sqrt{x^2 + 6x + 5} + \sqrt{x^2 + 2x + 1}}.$$

Step 3: Factor Out x

Factor x from each square root:

$$= \lim_{x \rightarrow \infty} \frac{4x + 4}{x \left(\sqrt{1 + \frac{6}{x} + \frac{5}{x^2}} + \sqrt{1 + \frac{2}{x} + \frac{1}{x^2}} \right)}.$$

Divide numerator and denominator by x :

$$= \lim_{x \rightarrow \infty} \frac{4 + \frac{4}{x}}{\sqrt{1 + \frac{6}{x} + \frac{5}{x^2}} + \sqrt{1 + \frac{2}{x} + \frac{1}{x^2}}}.$$

Step 4: Take the Limit

As $x \rightarrow \infty$, all terms involving $\frac{1}{x}$ and higher powers go to 0. Therefore,

$$\lim_{x \rightarrow \infty} \frac{4 + \frac{4}{x}}{\sqrt{1 + \frac{6}{x} + \frac{5}{x^2}} + \sqrt{1 + \frac{2}{x} + \frac{1}{x^2}}} = \frac{4}{1 + 1} = 2.$$

Final Answer

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