

Question 2

Evaluate the limit:

$$\lim_{x \rightarrow -\infty} \frac{3x^2 + 7x - 5}{\sqrt{x^4 + 2x^2 + 1}}.$$

Question 2 - Solution

We evaluate

$$\lim_{x \rightarrow -\infty} \frac{3x^2 + 7x - 5}{\sqrt{x^4 + 2x^2 + 1}}.$$

Step 1: Rewrite the denominator carefully

Note that

$$\sqrt{x^4 + 2x^2 + 1} = \sqrt{(x^2 + 1)^2} = |x^2 + 1|.$$

Since $x^2 + 1 > 0$ for all real x ,

$$|x^2 + 1| = x^2 + 1.$$

Thus, the expression becomes

$$\frac{3x^2 + 7x - 5}{x^2 + 1}.$$

Step 2: Divide by the highest power of x

Divide both numerator and denominator by x^2 :

$$\frac{3 + \frac{7}{x} - \frac{5}{x^2}}{1 + \frac{1}{x^2}}.$$

Step 3: Take the limit

As $x \rightarrow -\infty$,

$$\frac{7}{x} \rightarrow 0, \quad \frac{5}{x^2} \rightarrow 0, \quad \frac{1}{x^2} \rightarrow 0.$$

Therefore,

$$\lim_{x \rightarrow -\infty} \frac{3 + \frac{7}{x} - \frac{5}{x^2}}{1 + \frac{1}{x^2}} = \frac{3}{1} = 3.$$

Final Answer

$$\boxed{3}$$