

Question 1

Evaluate the limit:

$$\lim_{x \rightarrow \infty} \left(\sqrt{x^2 + 5x + 7} - \sqrt{x^2 + x} \right).$$

Question 1 - Solution

We evaluate

$$\lim_{x \rightarrow \infty} \left(\sqrt{x^2 + 5x + 7} - \sqrt{x^2 + x} \right).$$

This expression has the indeterminate form $\infty - \infty$. To resolve it, we multiply by the conjugate.

Step 1: Multiply by the Conjugate

$$= \lim_{x \rightarrow \infty} \left(\sqrt{x^2 + 5x + 7} - \sqrt{x^2 + x} \right) \cdot \frac{\sqrt{x^2 + 5x + 7} + \sqrt{x^2 + x}}{\sqrt{x^2 + 5x + 7} + \sqrt{x^2 + x}}.$$

This simplifies to

$$= \lim_{x \rightarrow \infty} \frac{(x^2 + 5x + 7) - (x^2 + x)}{\sqrt{x^2 + 5x + 7} + \sqrt{x^2 + x}}.$$

Step 2: Simplify the Numerator

$$(x^2 + 5x + 7) - (x^2 + x) = 4x + 7,$$

so the limit becomes

$$\lim_{x \rightarrow \infty} \frac{4x + 7}{\sqrt{x^2 + 5x + 7} + \sqrt{x^2 + x}}.$$

Step 3: Factor Out x

Factor x from each square root:

$$= \lim_{x \rightarrow \infty} \frac{4x + 7}{x \left(\sqrt{1 + \frac{5}{x} + \frac{7}{x^2}} + \sqrt{1 + \frac{1}{x}} \right)}.$$

Divide numerator and denominator by x :

$$= \lim_{x \rightarrow \infty} \frac{4 + \frac{7}{x}}{\sqrt{1 + \frac{5}{x} + \frac{7}{x^2}} + \sqrt{1 + \frac{1}{x}}}.$$

Step 4: Take the Limit

As $x \rightarrow \infty$,

$$\frac{7}{x} \rightarrow 0, \quad \frac{5}{x} \rightarrow 0, \quad \frac{7}{x^2} \rightarrow 0, \quad \frac{1}{x} \rightarrow 0.$$

Thus,

$$\lim_{x \rightarrow \infty} \frac{4 + \frac{7}{x}}{\sqrt{1 + \frac{5}{x} + \frac{7}{x^2}} + \sqrt{1 + \frac{1}{x}}} = \frac{4}{1 + 1} = 2.$$

Final Answer

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