

Question 8

Determine the limit:

$$\lim_{x \rightarrow \infty} \frac{6x^2 - x + 8}{\sqrt{x^4 + 3x^2 + 1}}$$

Question 8 - Solution

We are given:

$$\lim_{x \rightarrow \infty} \frac{6x^2 - x + 8}{\sqrt{x^4 + 3x^2 + 1}}$$

Step 1: Rewrite the denominator using properties of radicals:

$$\sqrt{x^4 + 3x^2 + 1} = x^2 \sqrt{1 + \frac{3}{x^2} + \frac{1}{x^4}}$$

So the entire expression becomes:

$$\frac{6x^2 - x + 8}{x^2 \sqrt{1 + \frac{3}{x^2} + \frac{1}{x^4}}} = \frac{x^2 (6 - \frac{1}{x} + \frac{8}{x^2})}{x^2 \sqrt{1 + \frac{3}{x^2} + \frac{1}{x^4}}}$$

Cancel x^2 :

$$= \frac{6 - \frac{1}{x} + \frac{8}{x^2}}{\sqrt{1 + \frac{3}{x^2} + \frac{1}{x^4}}}$$

Step 2: Let $x \rightarrow \infty$:

$$= \frac{6 - 0 + 0}{\sqrt{1 + 0 + 0}} = \frac{6}{1} = \boxed{6}$$