

Question 2

Evaluate the limit:

$$\lim_{x \rightarrow \infty} (\sqrt{x^2 + 5x} - x)$$

Question 2 - Solution

We are given:

$$\lim_{x \rightarrow \infty} (\sqrt{x^2 + 5x} - x)$$

This expression is of the indeterminate form $\infty - \infty$, so we need to rationalize:

Multiply by the conjugate:

$$(\sqrt{x^2 + 5x} - x) \cdot \frac{\sqrt{x^2 + 5x} + x}{\sqrt{x^2 + 5x} + x}$$

This gives:

$$\frac{(\sqrt{x^2 + 5x})^2 - x^2}{\sqrt{x^2 + 5x} + x} = \frac{x^2 + 5x - x^2}{\sqrt{x^2 + 5x} + x} = \frac{5x}{\sqrt{x^2 + 5x} + x}$$

Now divide numerator and denominator by x :

$$\frac{5}{\sqrt{1 + \frac{5}{x}} + 1}$$

As $x \rightarrow \infty$, $\frac{5}{x} \rightarrow 0$, so:

$$\frac{5}{\sqrt{1+0}+1} = \frac{5}{1+1} = \frac{5}{2}$$

Final Answer:

$$\boxed{\frac{5}{2}}$$