

Question 2

Evaluate the limit:

$$\lim_{x \rightarrow 0^+} \frac{\tan x}{x}.$$

Question 2 - Solution

We evaluate

$$\lim_{x \rightarrow 0^+} \frac{\tan x}{x}.$$

Method 1: Local Approximation

As $x \rightarrow 0$, the Taylor expansion of $\tan x$ is

$$\tan x = x + \frac{x^3}{3} + \cdots.$$

Thus,

$$\frac{\tan x}{x} = \frac{x + \frac{x^3}{3} + \cdots}{x} = 1 + \frac{x^2}{3} + \cdots.$$

As $x \rightarrow 0^+$, the higher-order terms approach 0, so

$$\lim_{x \rightarrow 0^+} \frac{\tan x}{x} = 1.$$

Method 2: Rewrite Using Sine and Cosine

Recall that

$$\tan x = \frac{\sin x}{\cos x}.$$

Then,

$$\frac{\tan x}{x} = \frac{\sin x}{x} \cdot \frac{1}{\cos x}.$$

Now evaluate each factor as $x \rightarrow 0^+$:

$$\lim_{x \rightarrow 0^+} \frac{\sin x}{x} = 1, \quad \lim_{x \rightarrow 0^+} \cos x = 1.$$

Therefore,

$$\lim_{x \rightarrow 0^+} \frac{\tan x}{x} = 1 \cdot \frac{1}{1} = 1.$$

Conclusion

$$\boxed{\lim_{x \rightarrow 0^+} \frac{\tan x}{x} = 1}$$