

Question 10

Evaluate the limit:

$$\lim_{x \rightarrow 0} \frac{1 - \cos(4x)}{x^2}$$

Question 10 - Solution

We are given the limit:

$$\lim_{x \rightarrow 0} \frac{1 - \cos(4x)}{x^2}$$

Step 1: Use the identity

$$1 - \cos(\theta) = 2 \sin^2\left(\frac{\theta}{2}\right)$$

Apply this identity to the numerator:

$$1 - \cos(4x) = 2 \sin^2(2x)$$

So the expression becomes:

$$\lim_{x \rightarrow 0} \frac{2 \sin^2(2x)}{x^2}$$

Step 2: Rewrite the sine term

$$= \lim_{x \rightarrow 0} 2 \cdot \left(\frac{\sin(2x)}{x}\right)^2$$

We can write:

$$\frac{\sin(2x)}{x} = \frac{\sin(2x)}{2x} \cdot 2 \rightarrow 1 \cdot 2 = 2 \quad \text{as } x \rightarrow 0$$

So:

$$\left(\frac{\sin(2x)}{x}\right)^2 \rightarrow 4 \Rightarrow 2 \cdot 4 = 8$$

Final Answer: 8