

Question 9

Evaluate the limit:

$$\lim_{x \rightarrow 0} \frac{\sin(3x)}{\tan(5x)}.$$

Question 9 - Solution

We evaluate

$$\lim_{x \rightarrow 0} \frac{\sin(3x)}{\tan(5x)}.$$

Step 1: Express tangent in terms of sine and cosine

Recall that

$$\tan(5x) = \frac{\sin(5x)}{\cos(5x)}.$$

Thus,

$$\frac{\sin(3x)}{\tan(5x)} = \frac{\sin(3x) \cos(5x)}{\sin(5x)}.$$

Step 2: Apply standard trigonometric limits

Rewrite the expression as

$$\frac{\sin(3x)}{3x} \cdot \frac{5x}{\sin(5x)} \cdot \frac{3}{5} \cdot \cos(5x).$$

Now take limits term by term as $x \rightarrow 0$:

$$\frac{\sin(3x)}{3x} \rightarrow 1, \quad \frac{5x}{\sin(5x)} \rightarrow 1, \quad \cos(5x) \rightarrow 1.$$

Therefore,

$$\lim_{x \rightarrow 0} \frac{\sin(3x)}{\tan(5x)} = 1 \cdot 1 \cdot \frac{3}{5} \cdot 1 = \frac{3}{5}.$$

$$\boxed{\frac{3}{5}}$$