

Question 8

Evaluate the limit:

$$\lim_{x \rightarrow \infty} (\sqrt{x^2 + 5x} - x)$$

Question 8 - Solution

We are asked to evaluate:

$$\lim_{x \rightarrow \infty} (\sqrt{x^2 + 5x} - x)$$

Step 1: Indeterminate Form

As $x \rightarrow \infty$, both $\sqrt{x^2 + 5x}$ and x grow large. The expression becomes $\infty - \infty$, an indeterminate form. We simplify.

Step 2: Multiply by the Conjugate

Multiply numerator and denominator by the conjugate:

$$\begin{aligned} & (\sqrt{x^2 + 5x} - x) \cdot \frac{\sqrt{x^2 + 5x} + x}{\sqrt{x^2 + 5x} + x} \\ &= \frac{(x^2 + 5x) - x^2}{\sqrt{x^2 + 5x} + x} = \frac{5x}{\sqrt{x^2 + 5x} + x} \end{aligned}$$

Step 3: Divide numerator and denominator by x

$$= \frac{5}{\sqrt{1 + \frac{5}{x}} + 1}$$

As $x \rightarrow \infty$, $\frac{5}{x} \rightarrow 0$, so:

$$\lim_{x \rightarrow \infty} \frac{5}{\sqrt{1 + \frac{5}{x}} + 1} = \frac{5}{1 + 1} = \frac{5}{2}$$

Answer: $\boxed{\frac{5}{2}}$