

Question 7

Evaluate the limit:

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+3x} - \sqrt{1+x}}{x}.$$

Question 7 - Solution

We evaluate

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+3x} - \sqrt{1+x}}{x}.$$

Step 1: Identify the Indeterminate Form

Substituting $x = 0$ gives

$$\frac{\sqrt{1} - \sqrt{1}}{0} = \frac{0}{0},$$

which is an indeterminate form. Algebraic simplification is required.

Step 2: Multiply by the Conjugate

Multiply the numerator and denominator by the conjugate of the numerator:

$$\frac{\sqrt{1+3x} - \sqrt{1+x}}{x} \cdot \frac{\sqrt{1+3x} + \sqrt{1+x}}{\sqrt{1+3x} + \sqrt{1+x}}.$$

This yields

$$\frac{(1+3x) - (1+x)}{x(\sqrt{1+3x} + \sqrt{1+x})} = \frac{2x}{x(\sqrt{1+3x} + \sqrt{1+x})}.$$

Cancel the common factor of x :

$$\frac{2}{\sqrt{1+3x} + \sqrt{1+x}}.$$

Step 3: Evaluate the Limit

Now substitute $x = 0$:

$$\lim_{x \rightarrow 0} \frac{2}{\sqrt{1+3x} + \sqrt{1+x}} = \frac{2}{1+1} = 1.$$

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