

Question 5

Evaluate the following limit:

$$\lim_{x \rightarrow 4} \frac{x - 4}{\sqrt{x} - 2}$$

Question 5 - Solution

We are given:

$$\lim_{x \rightarrow 4} \frac{x-4}{\sqrt{x}-2}$$

Step 1: Direct Substitution

Substitute $x = 4$:

$$\frac{4-4}{\sqrt{4}-2} = \frac{0}{0}$$

This is an indeterminate form, so we need to simplify.

Step 2: Use Algebraic Manipulation

We can rationalize the denominator. Multiply the numerator and denominator by the conjugate of the denominator:

$$\frac{x-4}{\sqrt{x}-2} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2} = \frac{(x-4)(\sqrt{x}+2)}{(\sqrt{x}-2)(\sqrt{x}+2)}$$

Simplify the denominator:

$$(\sqrt{x}-2)(\sqrt{x}+2) = x-4$$

So the expression becomes:

$$\frac{(x-4)(\sqrt{x}+2)}{x-4}$$

Cancel the common factor $x-4$ (note: valid as $x \rightarrow 4$, not at $x = 4$):

$$= \sqrt{x}+2$$

Now take the limit:

$$\lim_{x \rightarrow 4} \sqrt{x}+2 = \sqrt{4}+2 = 2+2 = \boxed{4}$$