

Question 3

Evaluate the limit:

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$$

Question 3 - Solution

We are given:

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$$

Step 1: Check for indeterminate form.

At $x = 0$, the expression becomes:

$$\frac{\sqrt{1+0} - 1}{0} = \frac{1 - 1}{0} = \frac{0}{0}$$

So we have the indeterminate form $\frac{0}{0}$. To resolve this, we rationalize the numerator.

Step 2: Multiply numerator and denominator by the conjugate of the numerator.

$$\frac{\sqrt{1+x} - 1}{x} \cdot \frac{\sqrt{1+x} + 1}{\sqrt{1+x} + 1} = \frac{(1+x-1)}{x(\sqrt{1+x}+1)} = \frac{x}{x(\sqrt{1+x}+1)}$$

Cancel x from numerator and denominator:

$$= \frac{1}{\sqrt{1+x} + 1}$$

Step 3: Take the limit of the simplified expression.

$$\lim_{x \rightarrow 0} \frac{1}{\sqrt{1+x} + 1} = \frac{1}{\sqrt{1+1} + 1} = \frac{1}{2}$$

Final Answer:

$$\boxed{\frac{1}{2}}$$