

Question 1

Evaluate the following limit, showing all necessary steps and justification:

$$\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 4} - 2}{x}$$

Question 1 - Solution

We are given:

$$\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 4} - 2}{x}$$

At $x = 0$, this becomes:

$$\frac{\sqrt{0+4}-2}{0} = \frac{2-2}{0} = \frac{0}{0}$$

This is an indeterminate form, so we must simplify the expression.

Step 1: Multiply numerator and denominator by the conjugate.

Multiply by:

$$\frac{\sqrt{x^2 + 4} + 2}{\sqrt{x^2 + 4} + 2}$$
$$\frac{\sqrt{x^2 + 4} - 2}{x} \cdot \frac{\sqrt{x^2 + 4} + 2}{\sqrt{x^2 + 4} + 2} = \frac{(\sqrt{x^2 + 4} - 2)(\sqrt{x^2 + 4} + 2)}{x(\sqrt{x^2 + 4} + 2)}$$

The numerator is a difference of squares:

$$(\sqrt{x^2 + 4})^2 - 2^2 = x^2 + 4 - 4 = x^2$$

So the entire expression becomes:

$$\frac{x^2}{x(\sqrt{x^2 + 4} + 2)} = \frac{x}{\sqrt{x^2 + 4} + 2}$$

Step 2: Evaluate the limit.

Now:

$$\lim_{x \rightarrow 0} \frac{x}{\sqrt{x^2 + 4} + 2} = \frac{0}{\sqrt{0+4} + 2} = \frac{0}{4} = 0$$

Final Answer:

$$\boxed{0}$$