

Question 10

Let the functions $f(x)$, $g(x)$, and $h(x)$ satisfy the inequalities:

$$g(x) \leq f(x) \leq h(x)$$

for all x near 2 (but not necessarily at 2), and suppose:

$$\lim_{x \rightarrow 2} g(x) = 4 \quad \text{and} \quad \lim_{x \rightarrow 2} h(x) = 4$$

Use this information to determine:

$$\lim_{x \rightarrow 2} f(x)$$

Explain the reasoning behind your conclusion.

Question 10 - Solution

We are told that for all x near 2:

$$g(x) \leq f(x) \leq h(x)$$

and

$$\lim_{x \rightarrow 2} g(x) = 4, \quad \lim_{x \rightarrow 2} h(x) = 4$$

This is a perfect scenario to apply the "Squeeze Theorem", which states:

If $g(x) \leq f(x) \leq h(x)$ near $x = a$, and

$$\lim_{x \rightarrow a} g(x) = \lim_{x \rightarrow a} h(x) = L$$

then

$$\lim_{x \rightarrow a} f(x) = L$$

Applying this here:

$$\lim_{x \rightarrow 2} f(x) = \boxed{4}$$

Conclusion: By the Squeeze Theorem, since both bounding functions tend to 4 as $x \rightarrow 2$, so must $f(x)$.