

Question 9

Let $\lim_{x \rightarrow 3} f(x) = 5$ and $\lim_{x \rightarrow 3} g(x) = -2$.

Evaluate the following limit using limit laws:

$$\lim_{x \rightarrow 3} \left(\frac{f(x)^2 + 3g(x)}{g(x) - f(x)} \right)$$

Justify each step using properties of limits.

Question 9 - Solution

We are given:

$$\lim_{x \rightarrow 3} f(x) = 5, \quad \lim_{x \rightarrow 3} g(x) = -2$$

We need to evaluate:

$$\lim_{x \rightarrow 3} \left(\frac{f(x)^2 + 3g(x)}{g(x) - f(x)} \right)$$

Step 1: Use limit laws in numerator and denominator.

Numerator:

$$\lim_{x \rightarrow 3} f(x)^2 = \left(\lim_{x \rightarrow 3} f(x) \right)^2 = 5^2 = 25$$

$$\lim_{x \rightarrow 3} 3g(x) = 3 \cdot \lim_{x \rightarrow 3} g(x) = 3 \cdot (-2) = -6$$

$$\text{So, } \lim_{x \rightarrow 3} (f(x)^2 + 3g(x)) = 25 - 6 = 19$$

Denominator:

$$\lim_{x \rightarrow 3} (g(x) - f(x)) = \lim_{x \rightarrow 3} g(x) - \lim_{x \rightarrow 3} f(x) = -2 - 5 = -7$$

Step 2: Final Answer

$$\lim_{x \rightarrow 3} \left(\frac{f(x)^2 + 3g(x)}{g(x) - f(x)} \right) = \frac{19}{-7} = \boxed{-\frac{19}{7}}$$