

Question 8

Assume the following limits exist:

$$\lim_{x \rightarrow -1} f(x) = 2, \quad \lim_{x \rightarrow -1} g(x) = 3$$

Evaluate the limit:

$$\lim_{x \rightarrow -1} [(f(x) - g(x))^2 + 4f(x)]$$

Use limit laws to justify each step.

Question 8 - Solution

We are given:

$$\lim_{x \rightarrow -1} f(x) = 2, \quad \lim_{x \rightarrow -1} g(x) = 3$$

We evaluate:

$$\lim_{x \rightarrow -1} [(f(x) - g(x))^2 + 4f(x)]$$

Step 1: Apply limit laws to each part

First, consider the difference:

$$\lim_{x \rightarrow -1} (f(x) - g(x)) = \lim_{x \rightarrow -1} f(x) - \lim_{x \rightarrow -1} g(x) = 2 - 3 = -1$$

Now square the result:

$$\lim_{x \rightarrow -1} (f(x) - g(x))^2 = (-1)^2 = 1$$

Next, evaluate the linear term:

$$\lim_{x \rightarrow -1} 4f(x) = 4 \cdot \lim_{x \rightarrow -1} f(x) = 4 \cdot 2 = 8$$

Step 2: Combine results

Using the sum rule for limits:

$$\lim_{x \rightarrow -1} [(f(x) - g(x))^2 + 4f(x)] = 1 + 8 = \boxed{9}$$