

Question 5

Let $f(x) = \sqrt{4x + 5}$ and $g(x) = \frac{x^2 + 2x - 3}{x - 1}$, and assume $\lim_{x \rightarrow 1} f(x) = 3$.

(a) Find $\lim_{x \rightarrow 1} g(x)$.

(b) Use the limit properties to find $\lim_{x \rightarrow 1} [f(x) \cdot g(x)]$.

Question 5 - Solution

We are given:

$$f(x) = \sqrt{4x + 5}, \quad g(x) = \frac{x^2 + 2x - 3}{x - 1}, \quad \lim_{x \rightarrow 1} f(x) = 3$$

(a) Find $\lim_{x \rightarrow 1} g(x)$:

First, factor the numerator of $g(x)$:

$$x^2 + 2x - 3 = (x + 3)(x - 1)$$

$$g(x) = \frac{(x + 3)(x - 1)}{x - 1}$$

For $x \neq 1$, cancel common factors:

$$g(x) = x + 3$$

So,

$$\lim_{x \rightarrow 1} g(x) = \lim_{x \rightarrow 1} (x + 3) = 4$$

Answer: 4

(b) Find $\lim_{x \rightarrow 1} [f(x) \cdot g(x)]$:

Use the limit product law:

$$\lim_{x \rightarrow 1} [f(x) \cdot g(x)] = \lim_{x \rightarrow 1} f(x) \cdot \lim_{x \rightarrow 1} g(x) = 3 \cdot 4 = 12$$

Answer: 12