

Question 4

Let

$$f(x) = \begin{cases} \frac{x^2 - 9}{x - 3}, & x \neq 3 \\ k, & x = 3 \end{cases}$$

- (a) Find $\lim_{x \rightarrow 3} f(x)$ using limit properties.
- (b) Determine the value of k such that $f(x)$ is continuous at $x = 3$.

Question 4 - Solution

We are given:

$$f(x) = \begin{cases} \frac{x^2 - 9}{x - 3}, & x \neq 3 \\ k, & x = 3 \end{cases}$$

(a) Find the limit:

First, factor the numerator:

$$x^2 - 9 = (x - 3)(x + 3)$$

So for $x \neq 3$, we can simplify:

$$f(x) = \frac{(x - 3)(x + 3)}{x - 3} = x + 3$$

Thus,

$$\lim_{x \rightarrow 3} f(x) = \lim_{x \rightarrow 3} (x + 3) = 6$$

Answer: $\boxed{\lim_{x \rightarrow 3} f(x) = 6}$

(b) Value of k for Continuity:

For $f(x)$ to be continuous at $x = 3$, we must have:

$$\lim_{x \rightarrow 3} f(x) = f(3)$$

From (a), the limit is 6. Therefore, set $f(3) = k = 6$

Answer: $\boxed{k = 6}$