

Question 3

Given the following limits:

$$\lim_{x \rightarrow 1} f(x) = 4, \quad \lim_{x \rightarrow 1} g(x) = -2, \quad \lim_{x \rightarrow 1} h(x) = 0$$

Define:

$$k(x) = \frac{f(x) \cdot h(x) + g(x)}{h(x)}$$

Evaluate $\lim_{x \rightarrow 1} k(x)$, or explain why it does not exist.

Assume the quotient denominator is nonzero in some deleted neighborhood of the approach point. Distinguish finite limits from signed infinite limits.

Question 3 - Solution

Step 1: Apply the product and sum laws

For the numerator,

$$\lim_{x \rightarrow 1} (f(x)h(x) + g(x)) = 4(0) + (-2) = -2.$$

The denominator satisfies $\lim_{x \rightarrow 1} h(x) = 0$.

Step 2: Determine whether a finite limit exists

The numerator tends to a nonzero number, so its magnitude stays bounded away from zero near $x = 1$. Since $h(x)$ is nonzero in a deleted neighborhood,

$$|k(x)| = \left| \frac{f(x)h(x) + g(x)}{h(x)} \right| \rightarrow \infty.$$

There is no finite limit.

Step 3: Examine the sign

The sign depends on the denominator. For example, choose $f(x) = 4$ and $g(x) = -2$, so that $k(x) = 4 - 2/h(x)$.

Denominator $h(x)$	Behavior of $k(x)$
$(x - 1)^2$	$\rightarrow -\infty$
$-(x - 1)^2$	$\rightarrow +\infty$
$x - 1$	opposite one-sided infinities

All three choices satisfy the given limits. Therefore,

A signed infinite limit is not determined.

Division by a zero limiting value is not a valid use of the quotient law.