

Question 1

Let the functions $f(x)$, $g(x)$, and $h(x)$ be defined by the following limits:

$$\lim_{x \rightarrow 2} f(x) = 4, \quad \lim_{x \rightarrow 2} g(x) = -3, \quad \lim_{x \rightarrow 2} h(x) = 0$$

Evaluate the following limit expressions using limit laws and properties:

(a) $\lim_{x \rightarrow 2} [2f(x) + 5g(x)]$

(b) $\lim_{x \rightarrow 2} \left[\frac{f(x) \cdot g(x)}{h(x)} \right]$

(c) Explain whether the given information determines a signed infinite limit in part (b).

Assume the quotient denominator is nonzero in some deleted neighborhood of the approach point. Distinguish finite limits from signed infinite limits.

Question 1 - Solution**(a) Sum and constant-multiple laws**

Using the given limits,

$$\begin{aligned}\lim_{x \rightarrow 2} [2f(x) + 5g(x)] &= 2 \lim_{x \rightarrow 2} f(x) + 5 \lim_{x \rightarrow 2} g(x) \\ &= 2(4) + 5(-3) = \boxed{-7}.\end{aligned}$$

(b) Quotient with a zero denominator limit

The product law gives

$$\lim_{x \rightarrow 2} f(x)g(x) = 4(-3) = -12, \quad \lim_{x \rightarrow 2} h(x) = 0.$$

The quotient law does not apply because the denominator tends to zero. Since the numerator tends to a nonzero value, its magnitude stays bounded away from zero near $x = 2$. Therefore,

$$\left| \frac{f(x)g(x)}{h(x)} \right| \rightarrow \infty.$$

The quotient has no finite limit.

(c) Is the sign of an infinite limit determined?

No. Take $f(x) = 4$ and $g(x) = -3$. Each denominator below satisfies the given limit and is nonzero for $x \neq 2$:

Denominator $h(x)$	Behavior of $-12/h(x)$
$(x-2)^2$	$\rightarrow -\infty$
$-(x-2)^2$	$\rightarrow +\infty$
$x-2$	opposite one-sided infinities

Thus more information about the sign of $h(x)$ is needed.

A signed infinite limit is not determined.