

Question 10

Consider the piecewise-defined function:

$$f(x) = \begin{cases} x^2 - 3x + 2, & \text{if } x < 1 \\ \sqrt{x+3} - 2, & \text{if } x \geq 1 \end{cases}$$

- (a) Find $\lim_{x \rightarrow 1^-} f(x)$.
- (b) Find $\lim_{x \rightarrow 1^+} f(x)$.
- (c) Determine whether $\lim_{x \rightarrow 1} f(x)$ exists.
- (d) Is f continuous at $x = 1$? Justify your answer.

Question 10 - Solution

We are given:

$$f(x) = \begin{cases} x^2 - 3x + 2, & \text{if } x < 1 \\ \sqrt{x+3} - 2, & \text{if } x \geq 1 \end{cases}$$

(a) Left-hand limit:

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (x^2 - 3x + 2) = 1^2 - 3(1) + 2 = 0$$

$$\boxed{\lim_{x \rightarrow 1^-} f(x) = 0}$$

(b) Right-hand limit:

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (\sqrt{x+3} - 2) = \sqrt{4} - 2 = 2 - 2 = 0$$

$$\boxed{\lim_{x \rightarrow 1^+} f(x) = 0}$$

(c) Two-sided limit:

Since both one-sided limits exist and are equal:

$$\boxed{\lim_{x \rightarrow 1} f(x) = 0}$$

(d) Continuity at $x = 1$:

We already have:

$$\lim_{x \rightarrow 1} f(x) = 0$$

Now evaluate the function at $x = 1$:

$$f(1) = \sqrt{1+3} - 2 = \sqrt{4} - 2 = 2 - 2 = 0$$

Since: $f(1)$ is defined , $\lim_{x \rightarrow 1} f(x) = f(1)$

We conclude:

$$\boxed{f \text{ is continuous at } x = 1}$$