

Question 9

Consider the piecewise-defined function:

$$f(x) = \begin{cases} \frac{\sin(x)}{x}, & \text{if } x < 0 \\ 2x + 1, & \text{if } x \geq 0 \end{cases}$$

- (a) Evaluate $\lim_{x \rightarrow 0^-} f(x)$.
- (b) Evaluate $\lim_{x \rightarrow 0^+} f(x)$.
- (c) Does $\lim_{x \rightarrow 0} f(x)$ exist? Explain.
- (d) Find $f(0)$, if it exists.

Question 9 - Solution

We are given:

$$f(x) = \begin{cases} \frac{\sin(x)}{x}, & \text{if } x < 0 \\ 2x + 1, & \text{if } x \geq 0 \end{cases}$$

(a) Left-hand limit as $x \rightarrow 0^-$:

Since $\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1$, and this applies as $x \rightarrow 0^-$ as well:

$$\lim_{x \rightarrow 0^-} f(x) = 1$$

(b) Right-hand limit as $x \rightarrow 0^+$:

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} (2x + 1) = 1$$

$$\lim_{x \rightarrow 0^+} f(x) = 1$$

(c) Two-sided limit:

Since both one-sided limits are equal:

$$\lim_{x \rightarrow 0} f(x) = 1$$

(d) Value at the point:

Use the definition for $x \geq 0$:

$$f(0) = 2(0) + 1 = 1$$

$$f(0) = 1$$