

Question 8

Let the function $f(x)$ be defined as:

$$f(x) = \begin{cases} \frac{x^2-4}{x-2}, & \text{if } x < 2 \\ 3x - 4, & \text{if } x \geq 2 \end{cases}$$

- (a) Find $\lim_{x \rightarrow 2^-} f(x)$.
- (b) Find $\lim_{x \rightarrow 2^+} f(x)$.
- (c) Determine whether $\lim_{x \rightarrow 2} f(x)$ exists.
- (d) Is f continuous at $x = 2$? Justify your answer.

Question 8 - Solution

We are given:

$$f(x) = \begin{cases} \frac{x^2-4}{x-2}, & \text{if } x < 2 \\ 3x-4, & \text{if } x \geq 2 \end{cases}$$

(a) Left-hand limit:

Note: $x^2 - 4 = (x - 2)(x + 2)$, so for $x \neq 2$,

$$\frac{x^2 - 4}{x - 2} = x + 2$$

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} (x + 2) = 4$$

$\lim_{x \rightarrow 2^-} f(x) = 4$

(b) Right-hand limit:

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (3x - 4) = 6 - 4 = 2$$

$\lim_{x \rightarrow 2^+} f(x) = 2$

(c) Two-sided limit:

The left-hand and right-hand limits are not equal:

$$\lim_{x \rightarrow 2^-} f(x) = 4 \neq 2 = \lim_{x \rightarrow 2^+} f(x)$$

$\lim_{x \rightarrow 2} f(x) \text{ does not exist}$

(d) Continuity at $x = 2$:

$\lim_{x \rightarrow 2} f(x)$ does not exist, $f(2) = 3(2) - 4 = 6 - 4 = 2$ (defined)

But since the limit does not exist, f is not continuous at $x = 2$.

$f \text{ is not continuous at } x = 2$