

Question 7

Consider the piecewise-defined function:

$$f(x) = \begin{cases} 3x + 1, & \text{if } x < 2 \\ x^2 - 4, & \text{if } x \geq 2 \end{cases}$$

- (a) Compute $\lim_{x \rightarrow 2^-} f(x)$.
- (b) Compute $\lim_{x \rightarrow 2^+} f(x)$.
- (c) Is $\lim_{x \rightarrow 2} f(x)$ defined? Explain.
- (d) What is the value of $f(2)$?

Question 7 - Solution

We are given:

$$f(x) = \begin{cases} 3x + 1, & \text{if } x < 2 \\ x^2 - 4, & \text{if } x \geq 2 \end{cases}$$

(a) Left-hand limit as $x \rightarrow 2^-$:

Use the definition for $x < 2$:

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} (3x + 1) = 3(2) + 1 = 7$$

$$\boxed{\lim_{x \rightarrow 2^-} f(x) = 7}$$

(b) Right-hand limit as $x \rightarrow 2^+$:

Use the definition for $x \geq 2$:

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (x^2 - 4) = 2^2 - 4 = 0$$

$$\boxed{\lim_{x \rightarrow 2^+} f(x) = 0}$$

(c) Two-sided limit:

$$\lim_{x \rightarrow 2^-} f(x) = 7, \quad \lim_{x \rightarrow 2^+} f(x) = 0$$

Since the one-sided limits are not equal:

$$\boxed{\lim_{x \rightarrow 2} f(x) \text{ does not exist}}$$

(d) Value of the function at 2:

Since $x = 2 \geq 2$, use the second case:

$$f(2) = 2^2 - 4 = 0$$

$$\boxed{f(2) = 0}$$