

Question 6

Let the function $f(x)$ be defined as:

$$f(x) = \begin{cases} \frac{\sin(\pi x)}{x}, & x < 0 \\ \frac{1}{x+1}, & x \geq 0 \end{cases}$$

- (a) Compute $\lim_{x \rightarrow 0^-} f(x)$
- (b) Compute $\lim_{x \rightarrow 0^+} f(x)$
- (c) Does $\lim_{x \rightarrow 0} f(x)$ exist? Explain.

Question 6 - Solution

We are given:

$$f(x) = \begin{cases} \frac{\sin(\pi x)}{x}, & x < 0 \\ \frac{1}{x+1}, & x \geq 0 \end{cases}$$

(a) Left-hand limit:

For $x < 0$, use:

$$\lim_{x \rightarrow 0^-} \frac{\sin(\pi x)}{x}$$

Using the identity $\lim_{x \rightarrow 0} \frac{\sin(\pi x)}{x} = \pi$ (from standard limit $\lim_{u \rightarrow 0} \frac{\sin(u)}{u} = 1$):

$$\lim_{x \rightarrow 0^-} \frac{\sin(\pi x)}{x} = \pi$$

$$\lim_{x \rightarrow 0^-} f(x) = \pi$$

(b) Right-hand limit:

For $x \geq 0$, use:

$$\lim_{x \rightarrow 0^+} \frac{1}{x+1} = \frac{1}{1} = 1$$

$$\lim_{x \rightarrow 0^+} f(x) = 1$$

(c) Two-sided limit:

Since:

$$\lim_{x \rightarrow 0^-} f(x) = \pi \quad \text{and} \quad \lim_{x \rightarrow 0^+} f(x) = 1$$

These are not equal, so the limit does not exist:

$$\lim_{x \rightarrow 0} f(x) \text{ does not exist}$$