

Question 5

Consider the function:

$$f(x) = \begin{cases} \frac{1}{x-3}, & x < 3 \\ \sqrt{x-3}, & x \geq 3 \end{cases}$$

- (a) Find $\lim_{x \rightarrow 3^-} f(x)$
- (b) Find $\lim_{x \rightarrow 3^+} f(x)$
- (c) Does $\lim_{x \rightarrow 3} f(x)$ exist? Justify your answer.

Question 5 - Solution

We are given:

$$f(x) = \begin{cases} \frac{1}{x-3}, & x < 3 \\ \sqrt{x-3}, & x \geq 3 \end{cases}$$

(a) Left-hand limit:

As $x \rightarrow 3^-$, we are approaching 3 from values less than 3. The expression is:

$$f(x) = \frac{1}{x-3}$$

As $x \rightarrow 3^-$, $x-3 \rightarrow 0^- \Rightarrow \frac{1}{x-3} \rightarrow -\infty$

$$\boxed{\lim_{x \rightarrow 3^-} f(x) = -\infty}$$

(b) Right-hand limit:

As $x \rightarrow 3^+$, the function becomes:

$$f(x) = \sqrt{x-3}$$

As $x \rightarrow 3^+$, $x-3 \rightarrow 0^+ \Rightarrow \sqrt{x-3} \rightarrow 0$

$$\boxed{\lim_{x \rightarrow 3^+} f(x) = 0}$$

(c) Does the limit exist?

Since

$$\lim_{x \rightarrow 3^-} f(x) = -\infty \quad \text{and} \quad \lim_{x \rightarrow 3^+} f(x) = 0$$

the two one-sided limits are not equal (and one diverges), so the limit $\lim_{x \rightarrow 3} f(x)$ does not exist.

$$\boxed{\text{Limit does not exist at } x = 3}$$