

Question 4

Let the function $f(x)$ be defined as:

$$f(x) = \begin{cases} x^2 - 4, & x < 1 \\ 2x + 1, & x \geq 1 \end{cases}$$

- (a) Find $\lim_{x \rightarrow 1^-} f(x)$
- (b) Find $\lim_{x \rightarrow 1^+} f(x)$
- (c) Does $\lim_{x \rightarrow 1} f(x)$ exist? Explain.

Question 4 - Solution

We are given:

$$f(x) = \begin{cases} x^2 - 4, & x < 1 \\ 2x + 1, & x \geq 1 \end{cases}$$

(a) Left-hand limit:

For $x < 1$, we use $f(x) = x^2 - 4$:

$$\lim_{x \rightarrow 1^-} f(x) = 1^2 - 4 = -3$$

$$\boxed{\lim_{x \rightarrow 1^-} f(x) = -3}$$

(b) Right-hand limit:

For $x \geq 1$, we use $f(x) = 2x + 1$:

$$\lim_{x \rightarrow 1^+} f(x) = 2(1) + 1 = 3$$

$$\boxed{\lim_{x \rightarrow 1^+} f(x) = 3}$$

(c) Two-sided limit:

$$\lim_{x \rightarrow 1^-} f(x) = -3 \quad \text{and} \quad \lim_{x \rightarrow 1^+} f(x) = 3$$

Since the one-sided limits are not equal, the overall limit does not exist:

$$\boxed{\lim_{x \rightarrow 1} f(x) \text{ does not exist}}$$