

Question 3

Let the function $f(x)$ be defined as:

$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2}, & x < 2 \\ x + 1, & x \geq 2 \end{cases}$$

- (a) Compute $\lim_{x \rightarrow 2^-} f(x)$
- (b) Compute $\lim_{x \rightarrow 2^+} f(x)$
- (c) Is $f(x)$ continuous at $x = 2$? Explain.

Question 3 - Solution

We are given:

$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2}, & x < 2 \\ x + 1, & x \geq 2 \end{cases}$$

(a) **Left-hand limit:** First, simplify the expression for $x < 2$:

$$f(x) = \frac{x^2 - 4}{x - 2} = \frac{(x - 2)(x + 2)}{x - 2} = x + 2 \quad \text{for } x \neq 2$$

So,

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} (x + 2) = 2 + 2 = 4$$

(b) **Right-hand limit:**

$$f(x) = x + 1 \quad \text{for } x \geq 2$$

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (x + 1) = 2 + 1 = 3$$

(c) **Continuity at $x = 2$:** Since

$$\lim_{x \rightarrow 2^-} f(x) = 4, \quad \lim_{x \rightarrow 2^+} f(x) = 3 \Rightarrow \lim_{x \rightarrow 2} f(x) \text{ does not exist}$$

And because the limit does not exist, the function is not continuous at $x = 2$.

Final Answer:

$\lim_{x \rightarrow 2^-} f(x) = 4, \quad \lim_{x \rightarrow 2^+} f(x) = 3, \quad \text{Not continuous at } x = 2$
