

**Question 2**

Let the function  $f$  be defined as follows:

$$f(x) = \begin{cases} 3x + 1, & x < 1 \\ x^2, & x \geq 1 \end{cases}$$

- (a) Compute  $\lim_{x \rightarrow 1^-} f(x)$
- (b) Compute  $\lim_{x \rightarrow 1^+} f(x)$
- (c) Does  $\lim_{x \rightarrow 1} f(x)$  exist? Justify your answer.

**Question 2 - Solution**

We are given:

$$f(x) = \begin{cases} 3x + 1, & x < 1 \\ x^2, & x \geq 1 \end{cases}$$

(a) **Left-hand limit:** For  $x < 1$ , use  $f(x) = 3x + 1$ :

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (3x + 1) = 3(1) + 1 = 4$$

(b) **Right-hand limit:** For  $x \geq 1$ , use  $f(x) = x^2$ :

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} x^2 = 1^2 = 1$$

(c) **Two-sided limit:** Since the left and right limits are not equal:

$$\lim_{x \rightarrow 1^-} f(x) = 4 \quad \text{and} \quad \lim_{x \rightarrow 1^+} f(x) = 1 \Rightarrow \lim_{x \rightarrow 1} f(x) \text{ does not exist}$$

**Final Answer:**

|                                                                                                                                        |
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| $\lim_{x \rightarrow 1^-} f(x) = 4, \quad \lim_{x \rightarrow 1^+} f(x) = 1, \quad \lim_{x \rightarrow 1} f(x) \text{ does not exist}$ |
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