

Question 1

Let the function f be defined as:

$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2}, & \text{if } x < 2 \\ x + 2, & \text{if } x \geq 2 \end{cases}$$

Evaluate:

$$\lim_{x \rightarrow 2^-} f(x) \quad \text{and} \quad \lim_{x \rightarrow 2^+} f(x)$$

State whether $\lim_{x \rightarrow 2} f(x)$ exists.

Question 1 - Solution

We analyze the left-hand and right-hand limits separately.

Left-hand limit:

For $x < 2$,

$$f(x) = \frac{x^2 - 4}{x - 2} = \frac{(x - 2)(x + 2)}{x - 2}$$

For $x \neq 2$, the $x - 2$ cancels:

$$f(x) = x + 2$$

So:

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} (x + 2) = 4$$

Right-hand limit:

For $x \geq 2$, we use the second piece:

$$f(x) = x + 2 \Rightarrow \lim_{x \rightarrow 2^+} f(x) = 4$$

Conclusion:

Since both one-sided limits exist and are equal:

$$\lim_{x \rightarrow 2} f(x) = \boxed{4}$$