

Question 9

Evaluate the limit:

$$\lim_{x \rightarrow 4} \frac{x - 4}{\sqrt{x} - 2}$$

Question 9 - Solution

We are given:

$$\lim_{x \rightarrow 4} \frac{x - 4}{\sqrt{x} - 2}$$

First, observe that:

$$x \rightarrow 4 \Rightarrow \text{numerator } x - 4 \rightarrow 0, \quad \text{denominator } \sqrt{x} - 2 \rightarrow 0$$

So, this is an indeterminate form $\frac{0}{0}$. We rationalize the denominator to resolve it.
Multiply numerator and denominator by the conjugate of the denominator:

$$\frac{x - 4}{\sqrt{x} - 2} \cdot \frac{\sqrt{x} + 2}{\sqrt{x} + 2} = \frac{(x - 4)(\sqrt{x} + 2)}{(\sqrt{x} - 2)(\sqrt{x} + 2)} = \frac{(x - 4)(\sqrt{x} + 2)}{x - 4}$$

Now cancel the common factor $x - 4$, assuming $x \neq 4$:

$$= \sqrt{x} + 2$$

Now take the limit:

$$\lim_{x \rightarrow 4} \sqrt{x} + 2 = \sqrt{4} + 2 = 2 + 2 = \boxed{4}$$