

Question 8

Evaluate the following limit:

$$\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{\sin(3x)}$$

Question 8 - Solution

We are asked to compute:

$$\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{\sin(3x)}$$

First, substitute $x = 0$ into the numerator:

$$e^{2(0)} - 1 = e^0 - 1 = 1 - 1 = 0.$$

Now substitute $x = 0$ into the denominator:

$$\sin(3 \cdot 0) = \sin(0) = 0.$$

So the expression has the indeterminate form

$$\frac{0}{0}.$$

Since both the numerator and denominator approach 0, we can use L'Hopital's Rule.

Differentiate the numerator:

$$\frac{d}{dx} (e^{2x} - 1) = 2e^{2x}.$$

Differentiate the denominator:

$$\frac{d}{dx} (\sin(3x)) = 3 \cos(3x).$$

Therefore,

$$\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{\sin(3x)} = \lim_{x \rightarrow 0} \frac{2e^{2x}}{3 \cos(3x)}.$$

Now substitute $x = 0$:

$$\frac{2e^{2(0)}}{3 \cos(3 \cdot 0)} = \frac{2e^0}{3 \cos(0)}.$$

Since

$$e^0 = 1$$

and

$$\cos(0) = 1,$$

we get

$$\frac{2e^0}{3 \cos(0)} = \frac{2(1)}{3(1)} = \frac{2}{3}.$$

Thus,

$$\boxed{\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{\sin(3x)} = \frac{2}{3}}$$