

Question 7

Let

$$f(x) = \begin{cases} \frac{\sin(5x)}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$$

Determine the value of k that makes $f(x)$ continuous at $x = 0$.

Question 7 - Solution

We are given a piecewise function:

$$f(x) = \begin{cases} \frac{\sin(5x)}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$$

To make $f(x)$ continuous at $x = 0$, we must have:

$$\lim_{x \rightarrow 0} f(x) = f(0) = k$$

We now compute:

$$\lim_{x \rightarrow 0} \frac{\sin(5x)}{x}$$

We use the identity:

$$\lim_{x \rightarrow 0} \frac{\sin(ax)}{x} = a$$

So:

$$\lim_{x \rightarrow 0} \frac{\sin(5x)}{x} = 5$$

Thus, for continuity at $x = 0$, we must have:

$$k = \boxed{5}$$

Conclusion: The function is continuous at $x = 0$ if $\boxed{k = 5}$.