

Question 3

Evaluate the limit, if it exists:

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+3x} - \sqrt{1-x}}{x}$$

Explain your reasoning clearly.

Question 3 - Solution

We are asked to evaluate:

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+3x} - \sqrt{1-x}}{x}$$

Direct substitution gives an indeterminate form $\frac{0}{0}$, so we simplify.

Multiply the numerator and denominator by the conjugate of the numerator:

$$\frac{\sqrt{1+3x} - \sqrt{1-x}}{x} \cdot \frac{\sqrt{1+3x} + \sqrt{1-x}}{\sqrt{1+3x} + \sqrt{1-x}}$$

This yields:

$$\frac{(1+3x) - (1-x)}{x(\sqrt{1+3x} + \sqrt{1-x})} = \frac{4x}{x(\sqrt{1+3x} + \sqrt{1-x})}$$

Cancel x :

$$\frac{4}{\sqrt{1+3x} + \sqrt{1-x}}$$

Now take the limit as $x \rightarrow 0$:

$$\frac{4}{\sqrt{1} + \sqrt{1}} = \frac{4}{2} = \boxed{2}$$