

Question 2

Let the function be defined as:

$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2}, & x < 2 \\ 3x - 2, & x \geq 2 \end{cases}$$

Determine whether $\lim_{x \rightarrow 2} f(x)$ exists. If it does, find its value. Justify your answer.

Question 2 - Solution

We are asked to evaluate:

$$\lim_{x \rightarrow 2} f(x)$$

where

$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2}, & x < 2 \\ 3x - 2, & x \geq 2 \end{cases}$$

Step 1: Evaluate the Left-Hand Limit

For $x < 2$,

$$f(x) = \frac{x^2 - 4}{x - 2} = \frac{(x - 2)(x + 2)}{x - 2}$$

As long as $x \neq 2$, we can cancel $x - 2$:

$$f(x) = x + 2$$

So:

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} (x + 2) = 4$$

Step 2: Evaluate the Right-Hand Limit

For $x \geq 2$, $f(x) = 3x - 2$, so:

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (3x - 2) = 3(2) - 2 = 6 - 2 = 4$$

Step 3: Compare One-Sided Limits

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x) = 4 \Rightarrow \lim_{x \rightarrow 2} f(x) = \boxed{4}$$

Conclusion: The limit exists and is equal to $\boxed{4}$.