

Question 1

Evaluate the following limit, and explain your reasoning clearly:

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{\sqrt{x} - 1}$$

Question 1 - Solution

We are asked to evaluate:

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{\sqrt{x} - 1}$$

This limit gives an indeterminate form $\frac{0}{0}$, so we must simplify.

Start by factoring the numerator:

$$x^2 - 1 = (x - 1)(x + 1)$$

Now rewrite the expression:

$$\frac{(x - 1)(x + 1)}{\sqrt{x} - 1}$$

To simplify, multiply numerator and denominator by the ****conjugate**** of the denominator:

$$\frac{(x - 1)(x + 1)}{\sqrt{x} - 1} \cdot \frac{\sqrt{x} + 1}{\sqrt{x} + 1}$$

This gives:

$$\frac{(x - 1)(x + 1)(\sqrt{x} + 1)}{x - 1}$$

Now cancel $x - 1$ (which is nonzero near $x = 1$):

$$(x + 1)(\sqrt{x} + 1)$$

Now substitute $x = 1$:

$$(1 + 1)(\sqrt{1} + 1) = 2 \cdot 2 = \boxed{4}$$