

Question 9

Let

$$f(x) = 5x - 1$$

Use the ε - δ definition of a limit to prove that:

$$\lim_{x \rightarrow 2} f(x) = 9$$

Question 9 - Solution

We are given:

$$f(x) = 5x - 1, \quad \lim_{x \rightarrow 2} (5x - 1) = 9$$

We want to show that:

$$\forall \varepsilon > 0, \exists \delta > 0 \text{ such that if } 0 < |x - 2| < \delta, \text{ then } |f(x) - 9| < \varepsilon$$

Step 1: Work with the expression $|f(x) - 9|$

$$|f(x) - 9| = |5x - 1 - 9| = |5x - 10| = 5|x - 2|$$

We want:

$$5|x - 2| < \varepsilon \Rightarrow |x - 2| < \frac{\varepsilon}{5}$$

Step 2: Choose δ

Let:

$$\delta = \frac{\varepsilon}{5}$$

Step 3: Conclusion

Then for all $\varepsilon > 0$, if $0 < |x - 2| < \delta$, we have:

$$|f(x) - 9| = 5|x - 2| < 5 \cdot \delta = \varepsilon$$

Therefore, by the ε - δ definition of the limit:

$$\boxed{\lim_{x \rightarrow 2} (5x - 1) = 9}$$