

Question 7

Let

$$f(x) = 3x - 1$$

Use the ε - δ definition of a limit to prove that:

$$\lim_{x \rightarrow 2} f(x) = 5$$

Question 7 - Solution

We are given the function:

$$f(x) = 3x - 1$$

and we are asked to prove:

$$\lim_{x \rightarrow 2} (3x - 1) = 5$$

using the ε - δ definition.

Step 1: Recall the definition.

We must show that for every $\varepsilon > 0$, there exists a $\delta > 0$ such that

$$0 < |x - 2| < \delta \quad \Rightarrow \quad |(3x - 1) - 5| < \varepsilon$$

Step 2: Simplify the expression inside the absolute value.

$$|(3x - 1) - 5| = |3x - 6| = 3|x - 2|$$

So we require:

$$3|x - 2| < \varepsilon \quad \Rightarrow \quad |x - 2| < \frac{\varepsilon}{3}$$

Step 3: Choose δ .

Let

$$\delta = \frac{\varepsilon}{3}$$

Step 4: Verification.

If $0 < |x - 2| < \delta = \frac{\varepsilon}{3}$, then:

$$|f(x) - 5| = |3x - 6| = 3|x - 2| < 3 \cdot \frac{\varepsilon}{3} = \varepsilon$$

Conclusion: By the ε - δ definition, we have:

$$\boxed{\lim_{x \rightarrow 2} (3x - 1) = 5}$$