

Question 6

Let

$$f(x) = \frac{1}{2}x + 4$$

Prove using the ε - δ definition of a limit that:

$$\lim_{x \rightarrow -2} \left(\frac{1}{2}x + 4 \right) = 3$$

Question 6 - Solution

We are given:

$$f(x) = \frac{1}{2}x + 4$$

and want to prove:

$$\lim_{x \rightarrow -2} \left(\frac{1}{2}x + 4 \right) = 3$$

using the formal ε - δ definition of a limit.

Definition: For every $\varepsilon > 0$, there exists a $\delta > 0$ such that

$$0 < |x + 2| < \delta \quad \Rightarrow \quad \left| \left(\frac{1}{2}x + 4 \right) - 3 \right| < \varepsilon$$

Step 1: Simplify the expression

$$\left| \left(\frac{1}{2}x + 4 \right) - 3 \right| = \left| \frac{1}{2}x + 1 \right| = \left| \frac{1}{2}(x + 2) \right| = \frac{1}{2}|x + 2|$$

So we want:

$$\frac{1}{2}|x + 2| < \varepsilon \quad \Rightarrow \quad |x + 2| < 2\varepsilon$$

Step 2: Choose δ

Let:

$$\delta = 2\varepsilon$$

Step 3: Conclusion

Then whenever $0 < |x + 2| < \delta$, we have:

$$\left| \left(\frac{1}{2}x + 4 \right) - 3 \right| = \frac{1}{2}|x + 2| < \frac{1}{2} \cdot \delta = \frac{1}{2}(2\varepsilon) = \varepsilon$$

Therefore, by the ε - δ definition,

$$\boxed{\lim_{x \rightarrow -2} \left(\frac{1}{2}x + 4 \right) = 3}$$