

Question 5

Let

$$f(x) = 5x - 3$$

Prove, using the ε - δ definition of a limit, that:

$$\lim_{x \rightarrow 2} (5x - 3) = 7$$

Question 5 - Solution

We are given:

$$f(x) = 5x - 3$$

We want to prove:

$$\lim_{x \rightarrow 2} (5x - 3) = 7$$

using the formal ε - δ definition of the limit.

Definition: For every $\varepsilon > 0$, there exists $\delta > 0$ such that

$$0 < |x - 2| < \delta \quad \Rightarrow \quad |(5x - 3) - 7| < \varepsilon$$

Step 1: Simplify the expression

$$|(5x - 3) - 7| = |5x - 10| = 5|x - 2|$$

So we want:

$$5|x - 2| < \varepsilon \quad \Rightarrow \quad |x - 2| < \frac{\varepsilon}{5}$$

Step 2: Choose δ

Let:

$$\delta = \frac{\varepsilon}{5}$$

Step 3: Conclusion

Then whenever $0 < |x - 2| < \delta$, we have:

$$|(5x - 3) - 7| = 5|x - 2| < 5\delta = \varepsilon$$

Therefore, by the ε - δ definition,

$$\boxed{\lim_{x \rightarrow 2} (5x - 3) = 7}$$