

Question 3

Let

$$f(x) = 5x - 1$$

Prove, using the ε - δ definition of a limit, that:

$$\lim_{x \rightarrow 2} f(x) = 9$$

Question 3 - Solution

We are given:

$$f(x) = 5x - 1, \quad \lim_{x \rightarrow 2} f(x) = 9$$

We will use the ε - δ definition of a limit:

Goal: For every $\varepsilon > 0$, find $\delta > 0$ such that:

$$0 < |x - 2| < \delta \quad \Rightarrow \quad |f(x) - 9| < \varepsilon$$

Step 1: Express $|f(x) - 9|$:

$$|f(x) - 9| = |5x - 1 - 9| = |5x - 10| = 5|x - 2|$$

We want:

$$5|x - 2| < \varepsilon \quad \Rightarrow \quad |x - 2| < \frac{\varepsilon}{5}$$

Step 2: Choose δ :

Let:

$$\delta = \frac{\varepsilon}{5}$$

Then if $0 < |x - 2| < \delta$, we have:

$$|f(x) - 9| = 5|x - 2| < 5\delta = \varepsilon$$

Conclusion:

Therefore, $\lim_{x \rightarrow 2} (5x - 1) = 9$ by the ε - δ definition.