

Question 1

Let

$$f(x) = 4x - 1$$

Use the ε - δ definition of the limit to prove that:

$$\lim_{x \rightarrow 2} f(x) = 7$$

Question 1 - Solution

We are given:

$$f(x) = 4x - 1, \quad \lim_{x \rightarrow 2} f(x) = 7$$

We will prove this using the ε - δ definition:

Goal: For every $\varepsilon > 0$, we must find $\delta > 0$ such that:

$$0 < |x - 2| < \delta \quad \Rightarrow \quad |f(x) - 7| < \varepsilon$$

Step 1: Write out $|f(x) - 7|$:

$$|f(x) - 7| = |4x - 1 - 7| = |4x - 8| = 4|x - 2|$$

Step 2: Make $4|x - 2| < \varepsilon$:

We can achieve this by choosing:

$$|x - 2| < \frac{\varepsilon}{4}$$

Step 3: Let $\delta = \frac{\varepsilon}{4}$.

Then, for all x such that $0 < |x - 2| < \delta$, we have:

$$|f(x) - 7| = 4|x - 2| < 4\delta = 4 \cdot \frac{\varepsilon}{4} = \varepsilon$$

Conclusion:

Therefore, $\lim_{x \rightarrow 2} f(x) = 7$ by the ε - δ definition.